SEQUENCES

(On this sheet we use the shorthand u_n to mean "the n^{th} term". So u_3 means "the third term", u_{n+1} means "the $n+1^{th}$ term, etc.)

Sequences defined using a formula for the n^{th} term
For each of the following sequences, write down the first 5 terms and the 20^{th} term.

1) $u_n = 3n - 2$ 2) $u_n = n^2 + n + 1$ 3) $u_n = 3^{n-1}$ 4) $u_n = 3^n - 1$ 5) $u_n =$ 5) $u_n = (n+1)(n+3)$

Finding the n^{th} term
Find the n^{th} term for each of the following sequences:

(a) 2, 5, 8, 11, 14, ... (b) 1, 5, 9, 13, 17, ... (c) 3, 9, 27, 81, 243, ... (d) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ ...
(e) 2, 8, 32, 128, 512, (f) 1, 3, 5, 7, 9, ... (g) 23, 18, 13, 8, 3, ... (h) 2, 4, 8, 16, 32, ...
(i) 1, 4, 9, 16, 25, ... (j) 5, 10, 20, 40, 80, (k) 1, 3, 9, 27, 81, ... (l) 4, 10, 18, 28, 40, …
(m) 1, 3, 6, 10, 15, … (n) 2, 6, 10, 14, 18, … (o) 3, 8, 15, 24, 35, … (p) 1, 5, 25, 125, 625, …
(q) 2, 5, 10, 17, 26, ... (r) 2, 6, 12, 20, 30, ... (s) 4, 12, 36, 108, 324 (t) 8, 4, 2, 1, , …

Sequences defined recursively

1) For each of the following sequences, write down the first 7 terms in the sequence.
(a) $u_1 = 3$; $u_n = u_{n-1} + 4$ (b) $u_1 = 2$; $u_n = 3u_{n-1}$ (c) $u_1 = 2$; $u_n = 2u_{n-1} - 1$
(d) $u_1 = 1$; $u_2 = 3$; $u_n = u_{n-1} + 2u_{n-2}$ (e) $u_1 = 4$; $u_2 = 1$; $u_n = u_{n-1} - u_{n-2}$ (f) $u_1 = 1$; $u_n = u_{n-1} + 2n - 1$

2) Describe each of the following sequences using a recursive formula like those in question 1. (As a hint, those terms which are given in order to start the sequence off are shown in bold type.)
(a) **3**, 6, 12, 24, 48, 96, … (b) **2**, 5, 11, 23, 47, 95, … (c) **1, 3**, 7, 17, 41, 99, …
(d) **2, 3**, 6, 10, 17, 28, 46, … (e) **1, 2, 3**, 6, 11, 20, 37, 68, … (f) **1, 2, 4**, 3, 1, -3, -6, -7, -4, 2, 5, 9 …
(g) **31**, 15, 7, 3, 1, … (h) **2**, 4, 7, 11, 16, 22, … (i) **1**, 2, 6, 24, 120, 720, 5040, …

Alternative Definitions
Below are 6 sequences defined by an expression for their nth term (1-6) and the same 6 sequences defined recursively (A-F). However, sequence 1 is NOT the same as sequence A, etc. For each of sequences 1-6, write down the first five terms, and state which recursive definition (A-F) matches this sequence.

1) $u_n = 2^n - 1$	A) $u_{n+1} = u_n + 3$;	$u_1 = 1$
2) $u_n = 3n + 1$	B) $u_{n+1} = u_n + 4$;	$u_1 = 3$
3) $u_n = 2^{n-1}$	C) $u_{n+1} = 2u_n + 1$;	$u_1 = 1$
4) $u_n = 3n - 2$	D) $u_{n+1} = u_n + 3$;	$u_1 = 4$
5) $u_n = n^2 + 1$	E) $u_{n+1} = 2u_n$;	$u_1 = 1$
6) $u_n = 4n - 1$	F) $u_{n+1} = u_n + 2n + 1$;	$u_1 = 2$

Method of Differences
Using the method of differences, or any other method, find the next two terms in each of the following sequences:

(a) 3, 7, 13, 21, 31, ... (b) 1, 3, 7, 15, 31, ... (c) 2, 5, 7, 12, 19, 31, ... (d) 2, 7, 14, 23, 34, ...
(e) 1, 3, 9, 27, 81, ... (f) 1, 3, 8, 19, 42, 89, ... (g) 4, 15, 36, 70, 120, 189, … (h) 1, 2, 5, 12, 29, 70, ...

Miscellaneous

1) Find the next two terms in the sequence: 1, 1, 3, 5, 11, 21, 43, …

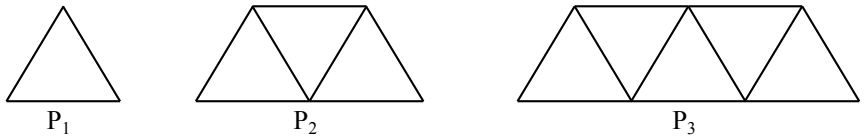
2) Find the next two terms in the sequence: 6, 24, 60, 120, 210, 336, ...

3) Find the 100th term in the sequence: 2, 6, 12, 20, 30, 42, 56, ...

4) Here is part of a sequence: 2 3 5 8 ...
(a) Suggest TWO DIFFERENT rules for this sequence, and in each case continue the sequence for 3 more numbers.
(b) Another rule to describe this sequence is to say that the 'nth' number in the sequence is given by the formula $\frac{n(n-1)}{2}+2$
Use this rule to find the 5th and 6th numbers in the sequence. Is this rule the same as one of your rules?
(c) Use the rule in (b) to find the 100th number in the sequence.

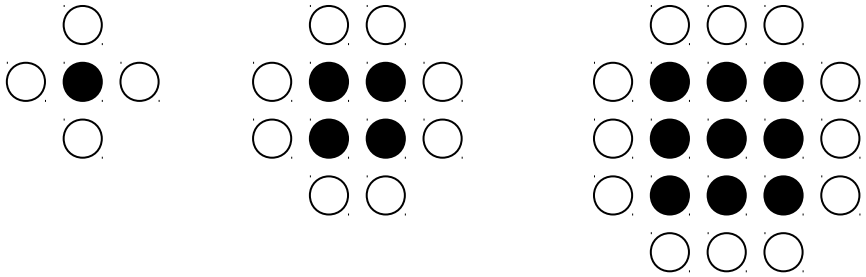
Sequences and Patterns

1) These are the first three in a series of patterns of lines:



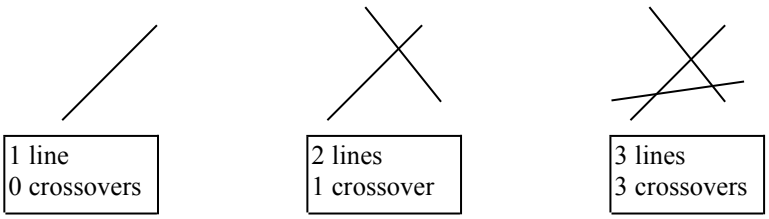
(a) Write down the number of lines in patterns P_1, P_2, P_3, P_4, and P_5.
(b) Write down a formula for the number of matchsticks in P_n.
(c) Show how you can explain your formula in (b) using diagrams.

2) A pattern consists of a square of black dots surrounded by 4 rows of white dots. Patterns of size 1, size 2 and size 3 are shown below.



(a) Draw the pattern of size 4
(b) How many black dots would there be in the pattern of size 9?
(c) How many white dots would there be in the pattern of size 7?
(d) If a pattern contained 44 white dots, how many black dots would it contain?
(e) If a pattern contained W white dots, write down a formula for the number of black dots. (B = ...)
(f) How many white dots are there in the pattern of size n?
(g) How many black dots are there in the pattern of size n?

3)



When a number of lines is drawn so that every line crosses every other line, the number of crossovers produced is as shown in the following table:

No of Lines:	1	2	3	4	5	6	…
No of crossovers	0	1	3	6	10	15	…

The recursive rule for this is $u_n = u_{n-1} + n - 1$. By referring to a diagram, explain **why** this rule works.

4)

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
…	…	…	…	…	…

The above pattern shows the natural numbers written in 6 columns. The number 16 is in the 3rd row and the 4th column.

(a) Which columns contain multiples of 3?
(b) In which column would the number 351 appear?
(c) In which row would 100 appear?
(d) Which columns can never contain multiples of 8? Give a reason for your answer.
(e) In which columns do all prime numbers (except 2 and 3) appear?
(f) Two of the columns will never contain a square number. Which are they?